

SENTENTIAL LOGIC USING SYMBOLS

SIMPLE PRESENTATION OF OUR SYMBOLS AND COMMON ENGLISH TRANSLATIONS

One amazing thing about thought and language is that despite the infinite number of statements we can link together in an extraordinarily large variety of ways, we can capture the relationships among them all by means of a few **logical connectives**. Here are our logical symbols, with some basic interpretive information:

SYMBOL	NAME OF SYMBOL	COMMON ENGLISH EQUIVALENTS	LOGICAL OPERATION
~	tilde	“not”; “it is not the case that” “it is false that”	Negation
&	ampersand	“while”; “and”; “nevertheless”; “also”; “moreover”; “but”; “yet”; “however” “still”; “additionally”; “furthermore”	Conjunction
V	wedge	“or”	Disjunction
→	arrow	“if...then”; “only if”; “given that”; “provided that”; “just in case”; “on condition that”; “sufficient condition for”; “necessary condition for”	Conditionality

A **simple sentence** of our symbolic language is a claim that is represented syntactically by a capital letter. For example, if I say, “Socrates went to the agora,” I can provide a **translation** of that sentence of English, which contains only one claim, as follows:

S: Socrates went to the agora.

The above is called a **scheme of abbreviation**. Without a scheme of abbreviation, no translation will have been given. Each claim in our translation must contain a unique capital letter, called a **constant**. You might think of it this way: “Since S refers to, ‘Socrates went to the agora’, every time I use S, it must refer *constantly* to the same **claim**. I stress that S refers to a *claim*, not a *name*.”

Now suppose we say something more complicated, and we want to translate *it* into a symbolic language. If it’s more complicated—if the sentence we are translating contains one or more of the above logical operators—then we call the result a **compound sentence**. For example:

Socrates went to the agora, and Xanthippe threw pee on him.

To translate this, we first provide a scheme of abbreviation, and this must include a separate capital letter for each *claim* in our compound sentence:

S: Socrates went to the agora.

X: Xanthippe threw pee on Socrates’s head.

Immediately above is our scheme of abbreviation. Notice that I did not write: “Xanthippe threw pee on him,” because the context makes it clear that the person on whom she threw the pee was Socrates. This should be your practice as well: read the sentence, grasp its meaning, and make sure your resulting symbolized sentences in your scheme of abbreviation include English sentences that are *standalone sentences*.

Now we are in a position to “get logical.” We are in a position to ask ourselves: “Given those two claims, given the *language* we have here presented to us, what are all the possible ways the *world* could be vis-à-vis that linguistic utterance?” First, we need to link the two claims together, so that we express in our symbolic language what the *relationship* is between them. Looking above at our logical connectives, it is this:

S & X

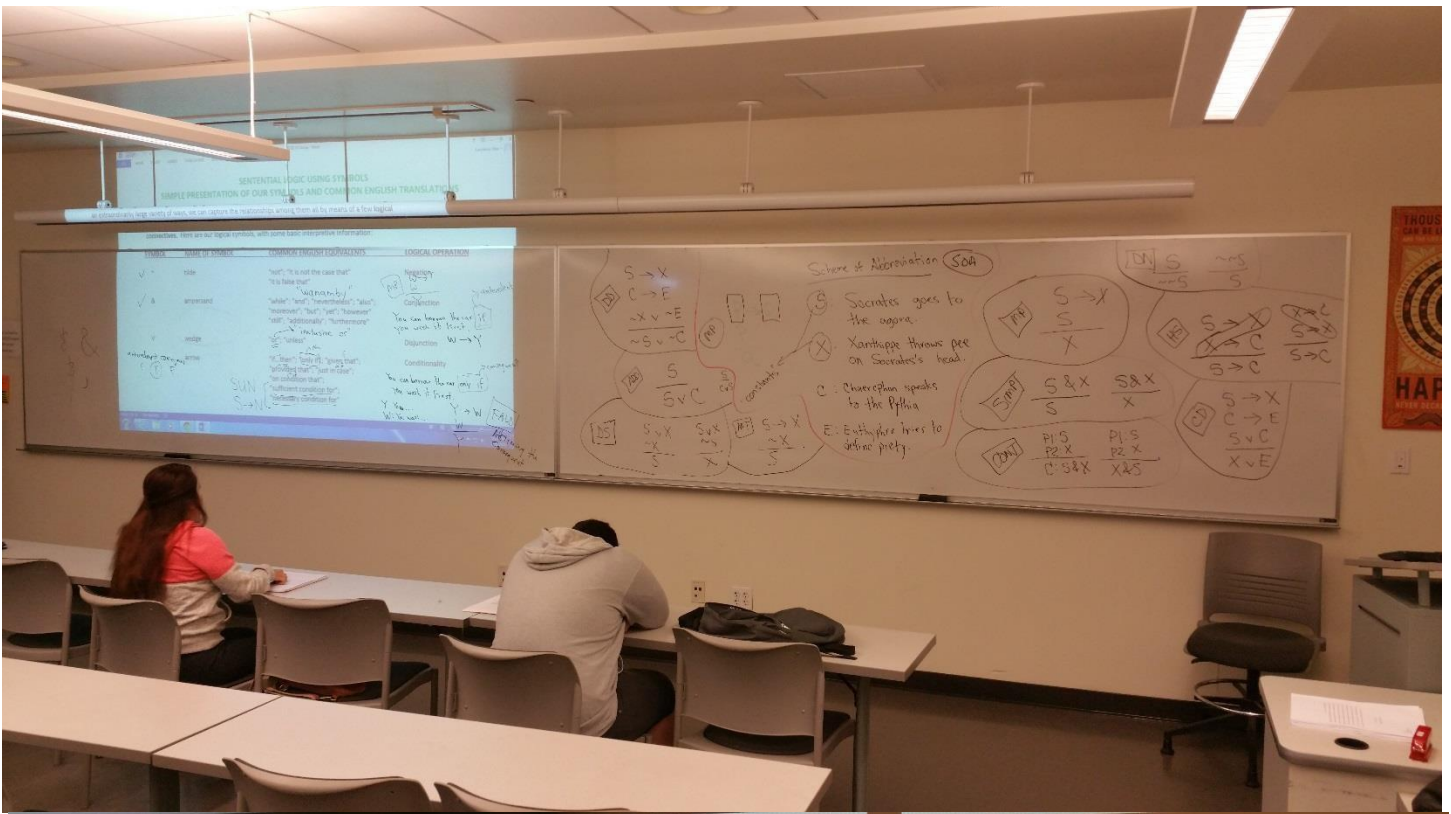
and that is our translation, because we have already provided our scheme of abbreviation further above.

This is the point at which our truth-tables come in handy, because we want a simple technique to keep track of all the ways things could possibly be, given that two claims have been made. Here is a truth-table that represents these possibilities. Remember that rows go across (horizontally), while columns go up and down (vertically):

S & X
T T T
T F F
F F T
F F F

The way to read this in English is this as follows. The first row under our translation “says”: “One possibility is that Socrates really did go to the agora, and Xanthippe really did throw pee on his head.” The second row under our translation says: “Another possibility is that Socrates really did go to the agora, but Xanthippe did not throw pee on his head.” The third row says: “A further possibility is that Socrates didn’t really go to the agora, but Xanthippe really did throw pee on his head.” The fourth row says: “The final possibility is that Socrates didn’t really go to the agora, and Xanthippe didn’t really throw pee on his head.” And when you think this over, you realize that these four possibilities truly are all of the logical possibilities about how the world could actually be, when it is compared with our original sentence of English.

So: Under what conditions is our sentence actually *true*?



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CONJUNCTION

Left conjunct	S & X	right conjunct
T	T	T
T	F	F
F	T	F
F	F	F

S	X
T	T
T	F
F	T
F	F

S
T
F

X
T
F

S	→	X
T	T	T
T	F	F
F	T	T
F	F	T

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Chaerophon spoke with the Pythia
 only if ^{then} [either Xanthippe threw pee on Socrates or Euthyphro tried to define piety].

SIA

C: Chaerophon spoke to the Pythia.
 X: Xanthippe threw pee on Socrates's head.
 E: Euthyphro tried to define piety.

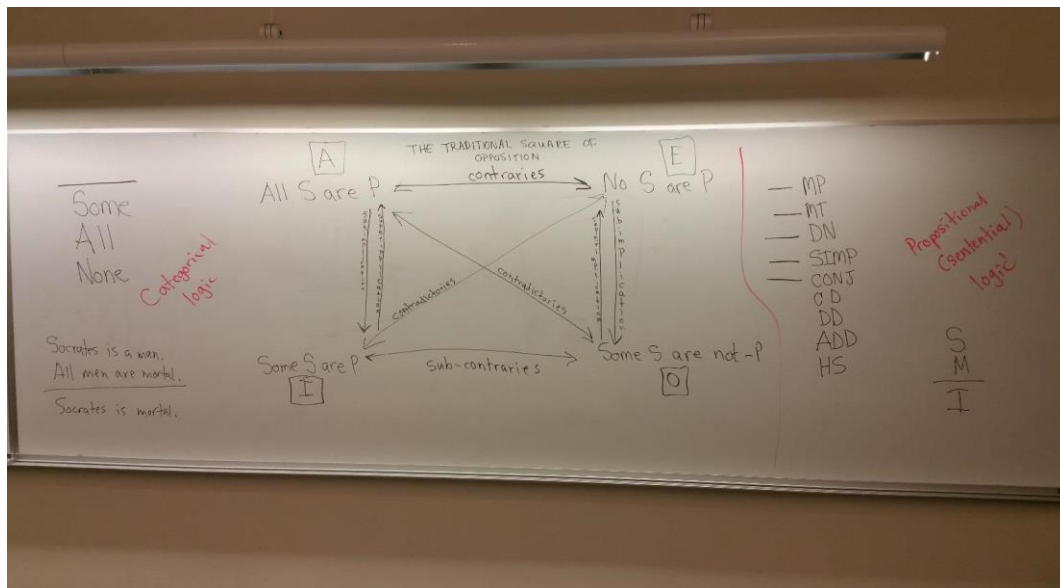
2^n | n = the number of constants
 $2^3 = 2 \times 2 \times 2 = 8$
 $2^4 = 2 \times 2 \times 2 \times 2$

C	X	E
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

This sentence is true in all logically \Diamond cases except when Chaerophon speaks with the Pythia, but Xanthippe doesn't throw pee on Socrates and Euthyphro doesn't try to define piety.

TRADITIONAL ARISTOTELIAN SQUARE OF OPPOSITION:

Can you define: contraries, contradictories, sub-contraries, superimplication and subimplication using this?



Contradictory statements are two statements that cannot have the same truth-value; so, if one of them is T, the other one is F.

Contrary statements are statements that cannot both be true, but may both be false.

Subcontrary statements are statements that can both be true, but cannot both be false.

We can only determine the truth value of **superimplication** when the I or O statement is false; if it is, then the corresponding A or E statement will be false as well.

We can only determine the truth value of **subimplication** if the A or E statement is true; if it is, then the corresponding I or O statement will be true as well.

The traditional Square of Opposition assumes that each category actually contains at least one member. It is therefore useful when considering most practical, pedestrian matters.

Many sentences of English can be “tweaked” so that they “fit” this pattern, without introducing any new meaning-bearing elements. This is one reason for thinking that we may organize the world, much but not all of the time, by means of categories. (This will strike some people as obviously true.)

NOTE: ‘unless’ is translated as the antecedent of a conditional negated. For example:

I’ll die unless I have air.

is translated as:

A: I have air.

D: I will die.

$\sim A \rightarrow D$